Double Difference Static GNSS

Jeff Faulkner
FIMarEST CMarSci CMarTech

FOST HM
(RN Hydrographic School)
Happy New Year

• I apologise in advance if you turned up to listen to Andy Waddington.
Who am I?
Relevant bits of the CV

• Played at, on and by the sea since before I could walk……always had an interest in the sciences.

• Surveyed in some exciting places with the RN. Retired in 08 and returned to the Hydro School as a part time civilian (Babcock) to teach on the Advanced Survey Course (Cat A & PG Dip).

• Subjects I cover: wave physics, Fourier analysis, least squares (thanks Rick), Kalman filtering, geodesy & control (inc astro, orbital mechanics, gravity, time & relativity), GNSS, tidal theory, analysis and offshore reference frames, singlebeam, sidescan, synthetic aperture and multi-pulse sonar.

• Nowadays - an aging surfer who has a part time hobby of hydrography.
Guide

• As this is a short notice, unrehearsed, quickly cobbled together piece can you please keep your questions the end.

• My “rule of thumb”. A presentation without a pretty picture and some squiggly maths is a presentation wasted.
“It is surprising how accurate a double differenced GPS static position can you get in just two epochs, but just how reliable is the solution?”

Discuss
Content

• A quick word on FOST HM
• A light refresh of our collective knowledge of:
  – precision and accuracy
  – propagation errors effecting GNSS signals
  – traditional Double Differenced static GNSS positioning.
• Look at some data and results from our students’ fieldwork over a number of courses.
• Generate and answer questions.
Knowledge refresh

- Accuracy and precision
- Ionospheric and tropospheric errors
- GPS pseudorange and phase equations
- Single and Double differencing
Accuracy

• As surveyors we strive for both accuracy and precision.

• Many people use the terms “accuracy” and “precision” interchangeably. However, for those in the surveying profession and other technical and scientific fields, the words have different meanings.
Accuracy

• “Accuracy” is the closeness a measurement is to the true value.
Precision

- “Precision” is the degree to which repeated measurements, under unchanged conditions, show the same results.
Knowledge refresh

- Accuracy and precision
- Ionospheric and tropospheric errors
- GPS pseudorange and phase equations
- Single and Double differencing
GNSS Errors - Ionosphere
GNSS Errors - Ionosphere

Global Ionospheric TEC Map

06/16/02 00:00 - 01:00 UT

Local Time (hours)

Geographic Latitude (deg)

TECU

GPS Receiver
## Ionospheric Range Error (m)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>1st order effect (1/f²)</th>
<th>2nd order effect (1/f³)</th>
<th>3rd order effect (1/f⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>32.5</td>
<td>0.036</td>
<td>0.002</td>
</tr>
<tr>
<td>L2</td>
<td>53.5</td>
<td>0.076</td>
<td>0.007</td>
</tr>
<tr>
<td>L1/L2</td>
<td>0.0</td>
<td>0.026</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Ionosphere free observable

\[
L_{IF} = \frac{f_1^2}{f_1^2 - f_2^2} L1 - \frac{f_2^2}{f_1^2 - f_2^2} L2
\]

For pseudo-range (units of metres) and carrier phase in cycles can be summarised:

\[
PR_{IF} = 2.545 \, PR_{L1} - 1.545 \, PR_{L2}
\]

\[
\Phi_{IF} = 2.545 \, \Phi_{L1} - 1.984 \, \Phi_{L2} \quad (units \ of \ L1)
\]
GNSS Errors - Troposphere

• Dual frequency will not help here.

• Two major delay effects of the troposphere
  – **Dry Atmosphere**
    – Approximates to 2.3m at zenith
    – Varies with local temperature and pressure (predictable)
    – Dry effect is variable by less than 1% in a few hours

  – **Wet Atmosphere**
    – Water Vapour effect $\approx$ 1 to 80 cm at zenith
    – 1/10th of the dry effect
    – Varies quickly 10-20% in a few hours and less predictable
    – Radiometer measurements 22 & 31 GHz enable more precise predictions but expensive and complicated
# Tropospheric Delay

<table>
<thead>
<tr>
<th>Elevation Angle</th>
<th>Dry Term (m)</th>
<th>Wet Term (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>2.3</td>
<td>0.2</td>
</tr>
<tr>
<td>30°</td>
<td>4.6</td>
<td>0.4</td>
</tr>
<tr>
<td>10°</td>
<td>13.0</td>
<td>1.2</td>
</tr>
<tr>
<td>5°</td>
<td>26.0</td>
<td>2.3</td>
</tr>
</tbody>
</table>
Knowledge refresh

- Accuracy and precision
- Ionospheric and tropospheric errors
- GPS pseudorange and phase equations
- Single and Double differencing
This can be expressed in a Pseudo-range Equation:

\[ PR = R + c\Delta t_a + c(\Delta t_u - \Delta t_s) \]
Pseudo-range Terms

\[ PR = R + c\Delta t_a + c(\Delta t_u - \Delta t_s) \]

- **R** – True Range
- **c** – Speed of Light
- **\( \Delta t_a \)** – Atmospheric delays
- **\( \Delta t_u \)** – User Clock Offset
- **\( \Delta t_s \)** – Satellite Clock Offset

This observation equation is a simplified version as it treats the satellite and receiver time frames together, whereas in reality they are different.
Time Frames

\[ T = \text{GPS time frame} \]
\[ t = \text{Satellite time frame} \]
\[ \tau = \text{Receiver time frame} \]

Therefore:

\[ T = t + \delta t = \tau + \delta \tau \]

Satellite clock offset
Receiver clock offset
Pseudo-range Terms - 2

If the time frames are considered separately, then a more rigorous pseudorange equation can be formed:

\[ PR^s_r (\tau_r) = \rho^s_r (T^s, T_r) - c[\delta \tau_r (\tau_r) - \delta t^s (t^s)] + d_{atm} \]

Where:

- \( PR^s_r \) = Pseudorange observed at receiver \( r \) in time frame of receiver
- \( \rho^s_r \) = Geometric range obtained from the true GPS time taken for the signal to leave satellite \( s \) and arrive at receiver \( r \)
- \( \delta \tau_r \) = Receiver clock offset for receiver \( r \) in receiver timeframe
- \( \delta t^s \) = Satellite clock offset for satellite \( s \) in satellite timeframe
- \( d_{atm} \) = Ionospheric and tropospheric delay biases.
Instead of observing a pseudo-range, the timing code is stripped off the signal and only the carrier wave phase is observed. The range therefore consists of:

- The Integer Ambiguity (N) i.e. an unknown number of wavelengths between satellite and receiver at the time the receiver first locks on to the satellite
- A carrier phase difference or fraction of a wavelength (\(\delta \lambda\)).

Measure: \(\delta \lambda\)
Determine: N
Carrier Phase Observation Equation - 1

- The Carrier Phase Equation is developed from the *pseudo-range* equation
- Is an expression that relates Carrier Phase with:
  - The geometric range
  - Receiver clock error
  - Satellite clock error
  - Integer ambiguity
  - Atmospheric delays.

\[
\phi_r^s = \frac{f}{c} \rho_r^s - f \left[ \delta \tau_r - \delta t^s \right] - N_r^s + d_{atm}
\]
Carrier Phase Observation Equation - 2

\[ \phi_r^s = \frac{f}{c} \rho_r^s - f \left[ \delta \tau_r - \delta t_r^s \right] - N_r^s + d_{atm} \]

Term consists of fractional part of a wavelength, change in integer number since lock-on and .... Receiver’s arbitrary ‘guess’ at the value of the integer ambiguity.

Geometric range between satellite & receiver

Rx Clock Offset
Satellite Clock Offset

Consists of a correction to the receiver’s arbitrary ‘guess’ at the integer ambiguity value i.e. the difference between the ‘true’ integer ambiguity and the receiver’s ‘guess’.
Single Difference

• If one SV and 2 receivers are considered then
• Because satellite 1 is common to both ranges the single difference removes satellite clock errors and atmospheric delays*

* Assumes delays are the same for both ranges.
Single Difference (A&B ⇒ 1)
Single Difference (A&B ⇒ 1)

\[
\varphi_A^1 = \frac{f}{c} \rho_A^1 - f \left[ \delta \tau_A - \delta t^1 \right] - N_A^1 + d_{atm}
\]

\[
\varphi_B^1 = \frac{f}{c} \rho_B^1 - f \left[ \delta \tau_B - \delta t^1 \right] - N_B^1 + d_{atm}
\]
Single Difference (A&B ⇒ 1)

\[ \varphi_{AB}^1 = \varphi_A^1 - \varphi_B^1 = \left[ \frac{f}{c} \rho_A^1 - \frac{f}{c} \rho_B^1 \right] + \left[ f\delta\tau_A - f\delta t^1 - f\delta\tau_B + f\delta t^1 \right] - \left[ N_A^1 - N_B^1 \right] + \left[ d_{atm} - d_{atm} \right] \]

\[ \varphi_{AB}^1 = \left[ \frac{f}{c} \rho_A^1 - \frac{f}{c} \rho_B^1 \right] - \left[ f\delta\tau_A - f\delta\tau_B \right] - \left[ N_A^1 - N_B^1 \right] \]
Single Difference (A&B ⇒ 2)

\[
\varphi_{AB}^1 = \left[ \frac{f}{c} \rho_A^1 - \frac{f}{c} \rho_B^1 \right] + \left[ f \delta \tau_A - f \delta \tau_B \right] - \left[ N_A^1 - N_B^1 \right]
\]

\[
\varphi_{AB}^2 = \left[ \frac{f}{c} \rho_A^2 - \frac{f}{c} \rho_B^2 \right] + \left[ f \delta \tau_A - f \delta \tau_B \right] - \left[ N_A^2 - N_B^2 \right]
\]
Double Difference

- As for the Single Difference but because the receivers A and B are common to both double differences, double-differencing removes the receiver clock error difference.
Double Difference \[ (A&B) \Rightarrow (1&2) \]
Double Difference

- Provides us with the observation equations from which to solve the integer ambiguities.
Double Difference - Calculations

• Software will produce 3 solutions:
  – A "Float" double difference
  – A "Fixed" double difference
  – A triple difference.
Double Difference - The Float Solution

The "Float" double difference solution solves for the ranges to several decimal places i.e. the equations are solved by a least squares process to give a straight mathematical solution.
Double Difference - The Fixed Solution

• The "Fixed" double difference solution takes the "Float" solution and deduces:

"This cannot be right ..... 
... N must be an integer"

• It therefore 'fixes' the integers by finding the best solution with integer values for N.
Double Difference - Solution

• The solution is presented as a vector i.e. the difference in the coordinates of the two ends of the baseline. \((\Delta X, \Delta Y, \Delta Z)\)

• A network therefore requires at least one known (fixed) station. However, current RN policy requires at least two.

• Occupation time

\[
T = \frac{B + E}{2}
\]

• Where ....
  • B = Baseline length in km between Rx
  • E = Rx recording Interval (in seconds).
Knowledge refresh

- Accuracy and precision
- Ionospheric and tropospheric errors
- GPS pseudorange and phase equations
- Single and Double differencing
Fieldwork

• Static calibration for the SMBs
  – RTK
  – PPK
  – Total Station

• PPK Coastlining exercise
  – Football pitch

• Static double difference obs
Static calibration for the SMBs
Football pitch PPK ex
Static double difference obs
The static DD question

- Using two Trimble R7 systems and the fixed antennas on FOST HM
- Set antenna height to 0.0m from base.
  - Antenna 1: record data at the following discrete intervals: 1, 2, 5, 10, 20, 30 mins, and 1, 2, and 6 hours,
  - Antenna 2: record data for a continuous period of 36 hours coincident with Ant 1 obs.
- Process:
  - Antenna 1 data with the Broadcast Ephemeris using single baseline only from PMTH and TAUT (OS Net).
  - Antenna 2 data using the Broadcast Ephemeris using a single short baseline (PMTH), a single long baseline (TAUT) and a single very long baseline (KIRK).
  - Antenna 2 data (24 hour segment) with both the Broadcast and the Rapid Ephemeris iaw HMOG criteria of at least two known stations.
- Comment on the quality of the derived positions.
Fieldwork – expected outcome

• A simple converging relationship of accuracy improving over time.
Course 20 Data – Sep 14
<table>
<thead>
<tr>
<th>Observation session length in minutes</th>
<th>Horizontal Precision in meters</th>
<th>Vertical Precision in meters</th>
<th>RMS in meters</th>
<th>Maximum PDOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Float solution)</td>
<td>0.022</td>
<td>0.029</td>
<td>0.001</td>
<td>1.916</td>
</tr>
<tr>
<td>2 (Fixed)</td>
<td>0.02</td>
<td>0.027</td>
<td>0.001</td>
<td>1.897</td>
</tr>
<tr>
<td>5</td>
<td>0.017</td>
<td>0.022</td>
<td>0.002</td>
<td>1.864</td>
</tr>
<tr>
<td>10</td>
<td>0.014</td>
<td>0.017</td>
<td>0.002</td>
<td>1.791</td>
</tr>
<tr>
<td>20</td>
<td>0.015</td>
<td>0.016</td>
<td>0.002</td>
<td>2.959</td>
</tr>
<tr>
<td>30</td>
<td>0.008</td>
<td>0.011</td>
<td>0.001</td>
<td>2.08</td>
</tr>
<tr>
<td>60</td>
<td>0.007</td>
<td>0.038</td>
<td>0.008</td>
<td>2.772</td>
</tr>
<tr>
<td>120</td>
<td>0.005</td>
<td>0.021</td>
<td>0.005</td>
<td>3.797</td>
</tr>
<tr>
<td>360</td>
<td>0.004</td>
<td>0.01</td>
<td>0.003</td>
<td>2.803</td>
</tr>
<tr>
<td>1440 (Antenna Two)</td>
<td>0.002</td>
<td>0.005</td>
<td>0.003</td>
<td>4.973</td>
</tr>
</tbody>
</table>
Course 20 Data – Sep 14

Antenna 1 Heights

Antenna Height (m)

Observation Period (Minutes)
**Analysis 20 min obs**

### Tracking Summary

<table>
<thead>
<tr>
<th>SV</th>
<th>11/09/2014 06:23:57</th>
<th>Duration: 00:21:44</th>
<th>Major interval: 00:01:00</th>
<th>11/09/2014 06:45:41</th>
</tr>
</thead>
<tbody>
<tr>
<td>G2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G12</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>G14</td>
<td></td>
<td></td>
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<td>G24</td>
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<td>G25</td>
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<tr>
<td>G29</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>G31</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Graphs**

1. **Graph 1:** Mean - 0.009 m, Std. Dev. - 0.005 m, Min. - 0.002 m, Max. - 0.020 m.
2. **Graph 2:** Mean - 0.029 m, Std. Dev. - 0.068 m, Min. - 0.011 m, Max. - 0.049 m.
Course 20 Data – Sep 14

Vertical Precision

Precision (m) vs Observation Period (Minutes) - Logarithmic Scale
Analysis 1 & 2 hr obs

• There is a height difference between the reference station at Plymouth and the antennae on Fitzroy Building of 103m.

• The troposphere is a considerable source of potential error and the proportion of water vapour can have a significant effect (Zenith Wet Delay - ZWD) when attempting to achieve mm level precision (Tekmon Geomatics 2014).

• In this case the vertical difference between stations, combined with the tendency of the Hamoze area, in which Fitzroy building is situated, to experience mist could reasonably be expected to have an impact on vertical precision.
Course 22 Data – Oct 15
Course 22 Data – Oct 2015

Single short baseline PMTH <5km - “Fixed Solution only”
Course 22 Data – Oct 2015

Length of Observation Vs Precision

<table>
<thead>
<tr>
<th></th>
<th>30 mins</th>
<th>1 hr</th>
<th>2 hr</th>
<th>6 hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>H Precision</td>
<td>0.023</td>
<td>0.021</td>
<td>0.014</td>
<td>0.009</td>
</tr>
<tr>
<td>V Precision</td>
<td>0.029</td>
<td>0.063</td>
<td>0.038</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Single baseline TAUT c100km - “Fixed Solution only”
Course 22 Data – Oct 2015

<table>
<thead>
<tr>
<th>SV</th>
<th>25/11/2015 11:11:44</th>
<th>Duration: 01:01:30 Major interval: 00:10:00</th>
<th>25/11/2015 12:13:14</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G3</td>
<td></td>
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<td></td>
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<td>G8</td>
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<td>G11</td>
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<td>G14</td>
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<td>G17</td>
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<td>G23</td>
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<td>G28</td>
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<td>G31</td>
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<td></td>
</tr>
<tr>
<td>G32</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Graphs showing data trends and anomalies](image-url)
Course 22 Data – Oct 2015

• Kirkwall baseline c 1000km only produced a “float” solution
  – battery
Course 22 Data – Oct 2015
Course 21 Data – Mar 2015

![Graph showing precision over time](image-url)
Course 19 Data – Jan 2014

Precision vs. observation time

- Horizontal precision
- Vertical precision

Precision

0.04
0.03
0.02
0.01
0

Observation time

16h 6h 2h 1h 30m 20m 10m 5m 2m 1m
Simple Summary

• You can get amazing results over very short periods of time
• But, accuracy of results in dual frequency DDGPS static positioning is not just about baseline length, masking angle, site occupation time and max PDOP
• You also need to consider:
  – baseline altitude differences (tropo) which can become significant in our maritime environment
  – satellite numbers – rising setting
• As well as multipath, power source, Naval transmissions, sandwiches, tea etc.
Questions?